## Greek mathematics <br> Theme : Géométrie de Collège

## L'usage de la calculatrice est autorise. Ce sujet comporte 1 page.

The Theorem of Thales, which states that an angle inscribed in a semicircle is a right angle, may have been learned by Thales while in Babylon but tradition attributes to Thales ( $624-548 \mathrm{BC}$ ) a demonstration of the theorem.
Another important figure in the development of Greek mathematics is Pythagoras of Samos (580-500 BC), who has commonly been given credit for discovering the Pythagorean theorem. Furthermore, many say that the Pythagoreans discovered most of the material in the first two books of Euclid's Elements, a collection of the mathematical knowledge of his age (around 300 BC ).

For a fourth mathematician, we could think of Eratosthenes ( $276-184 \mathrm{BC}$ ) who proposed a simple algorithm for finding prime numbers and also calculated the circumference of the Earth.

Adapted from http://en.wikipedia.org/wiki/Greek mathematics
and http://en. wikipedia. org/wiki/Eratosthenes

1. Read the first three lines of the text ending with 'a demonstration of the theorem'.
2. Explain what the text deals with and comment on it.

## Exercise

Let's consider a circle ( $r$ ) of diameter $[A B]$ and a third point $C$ on $\left(\|^{\sim}\right)$, such as $A B=5$ in and $B C=3$ in. Let's consider also $E$ on $[B C)$ and $F$ on $[B A)$, such as $(E F)$ is parallel to $(A C)$ and $B E=7$ in.

1. Make a rough drawing of the figure.
2. What sort of triangle is $A B C$ ? Explain.
3. Explain how you could calculate the length BF of triangle BEF.
4. Give two different ways of calculating EF.

## Origin of probability theory <br> Theme : Probabilités

## L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

Concepts of probability have been around for thousands of years, but the theory of probability did not arise as a branch of mathematics until the mid-17th century.
Many books on the history of probability start with: "A gambler named Chevalier de Mere presented two gambling problems to Blaise Pascal". Actually, Mere guessed that it was more likely to get at least one 6 during a total of four rolls of a die and the experience proved him right, then he tried to get at least one double 6 on twenty-four rolls of two dice. He asked his friend Blaise Pascal to help him solve the problem.
Pascal began to correspond with another mathematician, Pierre de Fermat, and their correspondence is the first documented evidence of the fundamental principles of the theory of probability.

1. Read the first two lines of the text ending with 'the mid-17th century'.
2. Explain what the text deals with and comment on it.

## EXERCISE

1. In the first trial, one fair die is cast.
a. If the die is cast only once, what is the probability not to roll a 6 ?
b. What is the probability of a 6 never appearing on 4 rolls of the die?
c. Explain how it's possible for a gambler to win money if the bet is to get at least one 6 during 4 rolls of the die.
2. Would you bet that at least one double 6 appears on 24 rolls of 2 fair dice?
3. Why?

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In the late 19th century there was a physicist named Alfred Binet. He was requested by the French government to find a way in which students of the public school system could be ordered properly according to their intellectual abilities. Binet devised a test that was composed of intellectual tasks.
The outcome of each test was an assigned "mental age" to each student. This dawned the beginning of what was later named the "IQ test". Many today claim that Binet was the inventor of the first intelligence test, which is completely contrary to what Binet said himself. Binet believed that intelligence was like that of beauty or love, which cannot be measured directly.

Extract from a blog entry by Paul-John Gagliano on Yahoo ! Voices, Sept. 2008

1. Start by reading the first paragraph of the text.
2. Explain what the text deals with and comment on it.

## EXERCISE

The score of an IQ test is calculated by dividing the proposed "mental age" by the student's chronological age, and then to multiply that by 100.

1. Write the formula that gives the IQ score from the mental age $M$ and the historical age $H$.
2. What is your IQ score if your mental abilities fit your age?
3. A 15 years old pupil has a mental age of 21 years. Compute his IQ score.
4. The same boy is credited 10 years later an IQ score of 84 . Calculate the increase of his mental age during that decade.
5. A female student is granted an IQ score of 120 . She shows today an intellectual level that is expected 4 years after. What is her age?

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## Ruler and compass construction <br> Theme : Géométrie de college

## L'usage de la calculatrice est autorisé. Ce sujet comporte $\mathbf{2}$ pages.

Compass-and-straightedge or ruler-and-compass construction is the construction of lengths, angles, and other geometric figures using only an idealized ruler and compass.
One of the chief purposes of Greek mathematics was to find exact constructions for various lengths; for example, the side of a pentagon inscribed in a given circle. But the Greeks could not find constructions for three problems:

- Squaring the circle: Drawing a square the same area as a given circle.
- Doubling the cube: Drawing a cube with twice the volume of a given cube.

- Trisecting the angle: Dividing a given angle into three smaller angles all of the same size.

For 2000 years people tried to find constructions within the limits set above, and failed.
All three have now been proven under mathematical rules to be impossible generally (angles with certain values can be trisected, but not all possible angles).

1. Start by reading the first paragraph of the text.
2. Explain what the text deals with and comment on it.

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## Exercise : Trisecting a line segment with Two Circles and Three Lines

1. Let $A B$ be a line segment of length 1 , describe step by step the following construction:

| Step 1 | Step 2 | Step 3 | Step 4 |
| :--- | :--- | :--- | :--- |
|  |  |  |  |

2. $\triangle A B G$ and $\triangle A B C$ are equilateral triangles because $A C=A B=C B=A G=B G=1$. Deduce that lines $A G$ and $C B$ are parallel.
3. $A$ is the midpoint of line segment $D C$ because $A C=A D=1$. In $\triangle B C D$, with the help of the previous question, show that $E$ is the midpoint of line segment $B D$.
4. As $A$ is the midpoint of $C D$, line segment $A B$ is a median of $A B C D$, and also line segment $C E$ is a median of $\triangle B C D$. Conclude that $A F=\frac{1}{3} A B$.

# BACCALAUREAT GENERAL <br> EPREUVE SPECIFIQUE DES SECTIONS EUROPEENNES 

Total eclipse<br>Theme: Astronomie

## L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page. 1

If the Moon's shadow sweeps across Earth's surface, then a total eclipse of the Sun is seen.
The total phase of a solar eclipse is very brief. It rarely lasts more than several minutes. Nevertheless, it is considered to be one of the most awe inspiring spectacles in all of nature. The sky takes on an eerie twilight as the Sun's bright face is replaced by the black disk of the Moon. Surrounding the Moon is a beautiful halo. This is the spectacular solar corona, a super heated plasma of two million degrees Celsius. The corona can only be seen during the few brief minutes the eclipse is total.

From the website «Mreclipse.com »

1. Read the first three lines of the text ending with 'spectacles in all of nature'.
2. Explain what the text deals with and comment on it.


## EXERCISE

1. The Earth-Moon distance is $369,000 \mathrm{~km}$. From caption 1, can you explain why it's an average?
2. During a total eclipse (see caption 2 ), the Sun (centre B) and the Moon (centre C) diameters seem to be the same from Earth (point O). The radius of the Moon is $1,737 \mathrm{~km}$ and the radius of the Sun is 695,500 km. Calculate the Earth-Sun distance OB.
3. Knowing that the speed of light is about $300,000 \mathrm{~km} / \mathrm{s}$, how long does it take for the light from the Sun to reach the Earth?

## The Chinese pond problem <br> Theme : Geometrie de collège

## L'usage de la calculatrice est autorise. Ce sujet comporte 1 page.

The Nine Chapters on the Mathematical Art - or Chiu Chang Suan Shu in Chinese - is one of the oldest Chinese mathematics book. It was composed by several generations of scholars from the 101 to the 2 na century BC. It lays out an approach to mathematics that centres on solving problems, which may be contrasted with the approach common to ancient Greek mathematicians, who tended to deduce propositions from an initial set of axioms. Some problems as the one given below suggest that the Chinese were aware of the Pythagorean theorem centuries before...Pythagoras! The Chinese of course didn't call it the Pythagorean theorem but the Gougu Theorem.

1. Read the first three lines ofthe text ending with '2nd century $B C$.
2. Explain what the text deals with and comment on it.

## Exercise

The following problem is the sixth in the ninth chapter of the Chiu Chang Suan Shu:
There is a circular pond of diameter 1 chang ( $=10$ chhih). A reed grows at its centre and extends 1 chhih above the water. If the reed is pulled to the side of the pond, its tip precisely touches the bank. What are the depth of the water and the length of the reed?

1. There is a right angled triangle in that situation. What are its dimensions?
2. 


a. This problem leads to an equation of unknown $x$. State this equation.
b. Solve this equation. Express the unknown depth and then the length of the reed.

# BACCALAUREAT GENERAL <br> EPREUVE SPECIFIQUE DES SECTIONS EUROPEENNES 

## Law of large numbers - Decision trees <br> Theme : Probabilité

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## Law of large numbers:

This is a statistical concept that says that the larger the sample population (or the number of observations) used in a test is, the more accurate the predictions of the behavior of that sample are, and smaller the expected deviation in comparisons of outcomes is.
As a general principle it means that, in the long run, the average (mean) of a long series of observations may be taken as the best estimate of the 'true value' of a variable.
In other words, what is unpredictable and chancy in case of an individual is predictable and uniform in the case of a large group.
In other words, a "law of large numbers" is one of several theorems expressing the idea that as the number of trials of a random process increases, the percentage difference between the expected and actual values goes to zero.
based on www.businessdictionary.com and www.mathworld. wolfram.com

1. Read the first three lines of the text ending with 'of outcomes is'.
2. Explain what the text deals with and comment on it.

## EXercise

Following the epidemic of influenza $B$ raging for several weeks in neighboring regions, a village launches a wide vaccination campaign, which allows $80 \%$ of the population to be vaccinated against the disease. In response to the controversy over the efficacy of the vaccine used, a study was conducted:
Only $5 \%$ of people vaccinated were infected;
And $50 \%$ of unvaccinated people were also contaminated.

1. Draw a weighted tree diagram.
2. What is the probability to cross randomly a contaminated inhabitant?
3. An association claims that if we randomly select a contaminated resident, there is more than one chance in three that he has been vaccinated previously. Is this consistent with the result of the study?

## BACCALAUREAT GENERAL <br> EPREUVE SPECIFIQUE DES SECTIONS EUROPEENNES

Motorcycling has become increasingly popular in recent years. It is popular among young people, and youthfulness and inexperience result in a higher than average risk of accident involvement, although many accidents involve motorcyclists in their thirties and forties and other factors also affect the risks of motorcycling. These risks are linked to a combination of factors associated with the motorcycle itself, the environment (including traffic conditions, road type, weather and road surface conditions), and the rider, while interactions with other road users are also a key factor.
This report has been commissioned to investigate road accidents involving motorcyclists in Great Britain. The analysis covers over 150,000 rider and passenger casualties on motorcycles from 2000 to 2006 . Of these, 94,000 were rider casualties, and the main focus of the analysis is on some 40,000 riders on bikes who were killed or seriously injured during this period.

Source : http://www.iam.org.uk/images/stories/downloads/Motorcycling/Motorcycle_casualties_-_main_report_Issue_3.pdf

1. Débuter l'entretien en lisant le texte à voix haute de «The analysis... » jusqu'à la fin.
2. Rendre compte du texte ci-dessus.

# BACCALAUREAT GENERAL <br> <br> EPREUVE SPECIFIQUE DES SECTIONS EUROPEENNES 

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## EXERCISE

The following data shows the stopping distance in dry conditions. The total stopping distance ( $D$ metres) is the sum of the distance travelled by the car before the motorist reacts (T metres) and the distance travelled by the car once the brakes have been applied ( $B$ metres).

| Speed (S km/h) | 0 | 20 | 40 | 60 | 80 | 100 | 120 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Thinking distance (T metres) | 0 | 6 | 12 | 18 | 24 | 30 | 40 |
| Breaking distance (B metres) | 0 | 4 | 16 | 36 | 64 | 100 | 144 |
| Total stopping distance (D metres) | 0 | 10 | 28 | 54 | 88 | 130 | 180 |

The distances T, B and D against S are plotted on the chart below.


1. On the chart above, match each curve with the corresponding distance.
2. It is known that a linear model describes the reacting distance:
a. There is a misprint in the table. One value of T does not follow this model. Which value is it?What should it be?
b. Find the formula for $T$ in terms of $S$ for the linear model.
3. Assuming that the formula for $D$ in terms of $S$ is $D=0.01><S 2$, and knowing that in town the speed limit is $50 \mathrm{~km} / \mathrm{h}$, use this model to find the total stopping distance at that speed.
4. To reduce serious injury the speed limit in some parts of towns is $30 \mathrm{~km} / \mathrm{h}$. By what percentage does this reduce the total stopping distance?

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| MATHEMATIQUES-ANGLAIS |

Analytic geometry, also known as coordinate geometry, or Cartesian geometry, is the study of geometry using a coordinate system and the principles of algebra and analysis. The Greek mathematician Menaechmus solved problems and proved theorems by using a method that had a strong resemblance to the use of coordinates and it has sometimes been maintained that he had introduced analytic geometry. Appollonius of Perga, in Determinate Section, dealt with problems in a manner that may be called an analytic geometry of one dimension; with the question of finding points on a line that were in a ratio to the others.
The eleventh century Persian mathematician Omar Khayyam helped to close the gap between numerical and geometric algebra with his geometric solution of the general cubic equations, but the decisive step came later with Descartes.

1. Start by reading the first paragraph of the text.
2. Explain what the text deals with and comment on it.

## EXERCISE

In an orthonormal coordinate system ( $O, I, J$ ), three points are given: $A(-3,0) ; B(6,3)$ and $C(1,8)$. The aim of this exercise is to calculate the coordinates of the circumcentre of the triangle $A B C$ labeled $K$.

1. Place $A, B$ and $C$ in the orthonormal coordinate system $(O, I, J)$ below.
2. Construct $K$ and draw the circumcircle of $\triangle A B C$.

Label ( $x, y$ ) the coordinates of $K$. Saying " $K$ is the centre of the circumcircle" is equivalent to saying:
$" K A^{2}=K B^{2}$ and $K B^{2}=K C^{2 "}$
a. Give an expression of $K A^{2}, K B^{2}$ and $K C^{2}$ as a function of $x$ and $y$.
b. Traduce then the equalities $K A 2=K B 2$ and $K B 2=K C 2$ in algebra.
3.
a. Deduce from question 2. b. that: $3 x+y=6$ and $-x+y=2$.
b. Calculate the coordinates of $K$.


L'usage de la calculatrice est autorise. Ce sujet comporte 2 pages, le matériel de géométrie est nécessaire.

## Carl Friedrich Gauss: Mathematical Prodigy

The German mathematician Carl Friedrich Gauss (1777-1855) was a prodigy child. As a ten-year-old student, Gauss was presented the following mathematical problem:
What is the sum of numbers from 1 to 100 ? While his fellow students were frantically calculating with paper and pencil, Gauss immediately envisioned that if he spread out the numbers through 50 from left to right, and the numbers 51 to 100 from right to left directly below the 1-50 numbers, each combination would add up to 101 $(1+100,2+99,3+98, \ldots)$. Since there were fifty sums, the answer would be: $101 \times 50=5050$. To the astonishment of everyone, young Carl got the answer ahead of everyone else, and computed it entirely in his mind. The teacher was so impressed that purchased the best available textbook on arithmetic and gave it to Gauss, stating, "He is beyond me, I can teach him nothing more."

Adapted from "Secrets of Mental Math "(Arthur Benjamin and Michael Shermer). 2006.

1. Start by reading the first seven lines of the text.
2. Explain what the text deals with and comment on it.

## EXERCISE

1. Guessing a rule.

Given : $15^{2}=225 \quad 25^{2}=625 \quad 35^{2}=1,225 \quad 45^{2}=2,025 \quad 55^{2}=3,025$.
a. What pattern can you see?
b. Explain how you compute mentally 652 using this pattern. What about 752 ?

## 2. Proof of the rule.

A whole number that ends in 5 can be written: 10a +5 where a is a whole number.
a. Prove the identity: $(10 a+5)^{2}=100 a(a+1)+25$.
b. What are the two digits that end the number $(10 a+5)^{2}$ ?
c. What is the number of hundreds in the number $(10 a+5)^{2}$ ?
d. What is your conclusion?

# BACCALAUREAT GENERAL <br> EPREUVE SPECIFIQUE DES SECTIONS EUROPEENNES 

## The hare and the tortoise <br> Theme : Fonctions

## L'usage de la calculatrice est autorise. Ce sujet comporte $\mathbf{2}$ pages.

One upon a time a hare was boasting ${ }^{1}$ of his own great speed. A tortoise smiled at the hare and replied, "Let us try a race. We shall run from here to the pond and the fox shall be the judge." The hare agreed and away they started together. True to his boasting the hare was out of sight in a moment. The tortoise jogged along with a slow, steady pace ${ }^{2}$, straight towards end of the course.
Having come nearly to the goal, the hare began to nibble at the young plants. After a while, he laid down for a nap ${ }^{3}$, saying: "The tortoise is me now. If he should go by, I can easily enough catch up ${ }^{4}$."
When the hare awoke, the tortoise was not in sight. Running as fast as he could hare found the fox congratulating the tortoise at the finish line.
(1) to boast: se vanter
(3) nap : sieste
(2) steady pace : pas regulier
(4) to catch up : rattraper

1. Start by reading the first paragraph of the text.
2. Explain what the text deals with and comment on it.
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## EXercise

The two distance-time graphs below show the race of the hare and the tortoise. Each one represents the distance (in meters) to the finish line. They are supposed to start at ten o'clock.


Answer the following questions as precisely as possible:

1. According to the two distance-time graphs :
a. What is the length, in total, of the race?
b. Which distance-time graph represents the race of the hare? Why?
c. When does the hare catch up to the tortoise ?
d. Work out what speed (in $\mathrm{km} / \mathrm{h}$ ) the hare was doing the first time it ran.
2. 

a. The distance-time graph of the hare is wrong. Explain why.
b. Draw another distance-time graph for the hare which can describe the situation of the fable

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| MATHEMATIQES-ANGLAIS |

## L'usage de la calculatrice est autorise. Ce sujet comporte 2 pages.

In 1698, Johann Bernoulli was the first to define a function as an analytic expression, he used the word "function" in an article on the solution to a problem involving curves and proposed the Greek letter $q>$ ( $p h i$ ) to be used as notation for a function. Euler introduced $f$ for function and brackets for $f(x)$. In 1748 , his definition of a function was: "A function of a variable quantity is an analytic expression composed in any way from this variable quantity and numbers or constant quantities."
Eventually, in 1755, Euler modified his definition in his Institutiones calculi differentialis : "If, therefore, x denotes a variable quantity, then all quantities which depend on $x$ in any way or are determined by itare called functions of it."

1. Read the first three lines of the text ending with 'notation for a function'.
2. Explain what the text deals with and comment on it.

## Exercise

A 4 star hotel manager wants to renew the corridor and stair carpets. High quality corridor carpet costs $£ 100$ per metre, or $£ 2400$ for a roll of 30 metres.

1. What is the best offer if the manager needs 25 metres of carpet?
2. The graph next page represents the price versus the length of the carpet. Justify its shape.
3. 

a. The manager wants to spend a total of $£ 4000$, what is the length of the carpet he can afford?
b. Same question if the manager has $£ 4800$.

